

Higher tier knowledge, skills and understanding

1. Number Structure and calculation

	Put positive and negative integers, decimals and fractions in order
	Use the symbols =, ≠, <, >, ≤, ≥
	+ , - , x and ÷ whole numbers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value
	Use BIDMAS/BODMAS
	Use and recognise prime numbers, factors, multiples, common factors, common multiples, HCF, LCM, prime factorisation
	List combinations systematically Including use of the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is m × n ways)
	Use and calculate positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5, estimate powers and roots of any given positive number. Calculate with roots, and with integer and fractional indices
	Use and calculate with standford form, $\times 10$
	Interchange fractions, decimals and percentages. Change recurring decimals into their corresponding fractions and vice versa
	Use multipliers and fractions in topics such as %, ratio and scale factor.
	Use standard units of mass, length, time, money and other measures((including standard compound measures, speed, density, pressure) using decimal quantities where appropriate. Change between units eg cm to m or c
	Estimate answers, round numbers to decimal places and significant figures
	Give simple error intervals, eg $25 \leq x < 35$, including upper and lower bounds
	Calculate exactly with fractions, surds and multiples of π ; simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators

2. Algebra

Notation, vocabulary and manipulation

	<p>use and interpret algebraic manipulation, including:</p> <ul style="list-style-type: none"> • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, • $\frac{a}{b}$ b in place of $a \div b$
	Substitute numbers into a formula
	Know the meaning and difference between expressions, equations, formulae, identities, inequalities, terms and factors
	<p>Simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions) by:</p> <ul style="list-style-type: none"> • collecting like terms • multiplying a single term over a bracket • taking out common factors • expanding products of two binomials • factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares and calculate roots, factorising quadratic expressions of the form $ax^2 + bx + c$ • simplifying expressions involving sums, products and powers, including the laws of indices <ul style="list-style-type: none"> • algebraic proofs
	Rearrange formulae to change the subject
	Show that algebraic expressions can be equal, to solve problems, often letting angles, areas and volumes be equal
	Use function machines - inputs and outputs. Interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function' (the use of formal function notation is expected)

Graphs

	Work with coordinates in all four quadrants, finding midpoints, completing shapes
	Plot straight lines, use the form $y = mx + c$ to identify parallel lines and perpendicular lines , find the equation of the line through two given points or through one point with a given gradient
	Identify and interpret roots, intercepts, turning points of x^2 graphs and turning points by completing the square
	Recognise, sketch and interpret graphs of <ul style="list-style-type: none"> • straight line graphs,

	<ul style="list-style-type: none"> • x^2 graphs • x^3 graphs, • $\frac{1}{x}$ • real life graphs - distance, speed and acceleration • exponential functions $y = k^x$ for positive values of k, • the trigonometric functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size • transform graphs
	Calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts (this does not include calculus)
	Recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point

Solving equations and inequalities

	Solve linear equations in one unknown algebraically, including unknown on both sides
	Solve two simultaneous equations in two variables, set the equations up from a scenario eg adult and children tickets, cups of tea and cakes
	Solve linear inequalities in one variable ($<$, $>$, \leq , \geq), and show inequalities on a number line, or write an inequality from a number line
	Solve quadratics by factorising, completing the square and using the quadratic formula.
	Find approximate solutions to equations numerically using iteration, finding the inverse function
	Solve linear inequalities in one or two variable(s), and quadratic inequalities in one variable; represent the solution set on a number line, using set notation and on a graph

Sequences

	Generate terms of a sequence from either terms eg 2, 6, 20, 14 or nth rule - $4n-2$
	Recognise and use sequences of different sets of numbers <ul style="list-style-type: none"> • Triangular 1, 3, 6, 10, 15 • Square 1, 4, 9, 16, 25 and cube numbers 1, 8, 64, 125

	<ul style="list-style-type: none"> • Fibonacci type sequences 0, 1, 1, 2, 3, 5, 8 or a, b, a+b, a+2b, 2a+3b • Simple geometric progressions eg 2^n, $2^0=1$, $2^1=2$, $2^2=4$, $2^3=8$ • Sequences including surds
	Find the nth term for a quadratic sequence


3 Ratio, proportion and rates of change

	Use scale factors, scale diagrams and maps
	Express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
	Use ratio notation, including reduction to simplest form
	Divide quantities into a ratio of it's parts from real life scenarios
	<ul style="list-style-type: none"> • Define percentage as 'number of parts per hundred'; • interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; • express one quantity as a percentage of another; compare two quantities using percentages; • work with percentages greater than 100%; • solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
	Solve problems involving direct and inverse proportion - knowing the graphs
	Compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
	Understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$
	Interpret equations that describe direct and inverse proportion
	Write equations for direct and indirect proportion
	Interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts (this does not include calculus)

4 Geometry and measures

Properties and constructions

	<p>Use conventional terms and notation:</p> <ul style="list-style-type: none"> • points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, • polygons, regular polygons and polygons with reflection and/or rotation symmetries • use the standard conventions for labelling and referring to the sides and angles of triangles • draw diagrams from written description
	<p>Use the standard ruler and compass constructions</p> <ul style="list-style-type: none"> • perpendicular bisector of a line segment, • constructing a perpendicular to a given line from/at a given point, • bisecting a given angle <p>Use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line</p>
	<p>Apply the properties of angles</p> <ul style="list-style-type: none"> • at a point, • angles at a point on a straight line, • vertically opposite angles • understand and use alternate and corresponding angles on parallel lines • derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
	<p>Derive and apply the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language</p>
	<p>Use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)</p>
	<p>Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proof</p>
	<p>Identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional and negative scale factors). Describe single transformations from combined transformations. Apply ideas of scale factors to areas volumes of similar shapes L. A. V.</p>
	<p>Identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment. Apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results</p>

	Construct and interpret plans and elevations of 3D shapes
	Measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
	<p>Know and apply formulae to calculate:</p> <ul style="list-style-type: none"> • area of triangles, • parallelograms, • trapezia; • volume of cuboids and other right prisms (including cylinders) • circumference of a circle = $2\pi r = \pi d$, • area of a circle = πr^2; • perimeters of 2D shapes, including circles; • areas of circles and composite shapes; • surface area and volume of spheres, pyramids, cones and composite solids • calculate arc lengths, angles and areas of sectors of circles
	<p>Know the formulae for:</p> <ul style="list-style-type: none"> • Pythagoras' theorem $a^2 + b^2 = c^2$, <div style="text-align: center;">  </div> <ul style="list-style-type: none"> • trigonometric ratios, apply them to find angles and lengths in right-angled triangles in two-dimensional figures • know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90°; • know the exact value of $\tan \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ$ and 60° • know and apply the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ • cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles
	Describe translations as 2D vectors, apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors. Use vectors to construct geometric arguments and proofs

5 Probability

	Record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
	Apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
	Relate relative expected frequencies to theoretical probability, using appropriate

	language and the 0-1 probability scale eg impossibly to certain
	Know that probabilities add up to 1
	Know a larger trail gives a better probability, closer to theoretical answers
	Use tables, grids, Venn diagrams and tree diagrams to solve probability questions
	Construct and use space diagrams for two events
	Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumption. Replacement and non-replacement.
	Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams

6 Statistics

	Infer properties of populations or distributions from a sample, while knowing the limitations of sampling
	Interpret and construct tables, charts and diagrams- <ul style="list-style-type: none"> ● frequency tables ● bar charts ● pie charts ● pictograms for categorical data ● vertical line charts for ungrouped discrete numerical data ● tables and line graphs for time series data ● scatter graphs - recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of so doing ● Histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use, including box plots and quartiles and inter-quartile ranges.
	Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: <ul style="list-style-type: none"> ● appropriate graphical representation involving discrete, continuous and grouped data ● appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)